Integration

Formula

①	$\int 0\,dx = c$	
②	$\int k\,dx = kx + c$	
③	$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$	$\int k(ax+b)^n dx = \frac{k(ax+b)^{n+1}}{a(n+1)} + c$
④	$\int e^x dx = e^x + c$	$\int e^{ax+b} dx = \frac{1}{a}e^{ax+b} + c$
⑤	$\int \frac{1}{x} dx = \ln\lvert x\rvert + c$	$\int \frac{1}{ax+b} dx = \frac{1}{a}\ln\lvert ax+b\rvert + c$
⑥	$\int \cos x\,dx = \sin x + c$	$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + c$
⑦	$\int \sin x\,dx = -\cos x + c$	$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + c$
⑧	$\int \sec^2 x\,dx = \tan x + c$	$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + c$
⑨	$\int \operatorname{cosec}^2 x\,dx = -\cot x + c$	$\int \operatorname{cosec}^2(ax+b)\,dx = \frac{1}{a}\cot(ax+b) + c$
⑩	$\int \sec x\tan x\,dx = \sec x + c$	$\int \sec(ax+b)\tan(ax+b)\,dx = \frac{1}{a}\sec(ax+b) + c$
⑪	$\int \operatorname{cosec} x\cot x\,dx = -\operatorname{cosec} x + c$	$\int \operatorname{cosec}(ax+b)\cot(ax+b)\,dx = -\frac{1}{a}\operatorname{cosec}(ax+b) + c$
⑫	$\int \frac{1}{1+x^2} dx = \tan^{-1}x + c$	$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
		$\int \frac{1}{1+(ax+b)^2} dx = \frac{1}{a}\tan^{-1}(ax+b) + c$
⑬	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}x + c$	$\int \frac{1}{\sqrt{1-(ax+b)^2}} dx = \frac{1}{a}\sin^{-1}(ax+b) + c$

Integration by substitution

Direct substitution : Ex) $\int \frac{1}{x}\ln x\,dx$

Let $u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$, $du = \frac{1}{x}dx$

$= \int u\,du$

$= \frac{u^2}{2} + c$

$= \frac{1}{2}(\ln x)^2 + c$

For definite integral, limits change

Reverse substitution : Ex) $\int \frac{1}{x}\ln x\,dx$

$= \int \ln x\,d(\ln x)$

$= \frac{1}{2}(\ln x)^2 + c$

For definite integral, limits do not change

Integration by parts :

Ex) $\int x e^x dx$

$= uv - \int v\,du$

$= xe^x - \int e^x dx$

$= xe^x - e^x + c$

Let $u = x$, $\frac{dv}{dx} = 1$, $du = dx$; $v = \int e^x dx$, $v = e^x$

Integration by partial fraction

Ex) $\int \frac{2}{(1+x)(1-x)} dx$

$= \int \frac{1+x+1-x}{(1+x)(1-x)} dx$

$= \int \frac{1+x}{(1+x)(1-x)} + \frac{1-x}{(1+x)(1-x)} dx$

$= \int \frac{1}{1-x} + \frac{1}{1+x} dx$

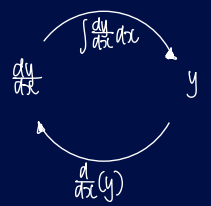
$= \frac{\ln\lvert 1-x\rvert}{-1} + \ln\lvert 1+x\rvert + c$

$= \ln\lvert 1+x\rvert - \ln\lvert 1-x\rvert + c$

$= \ln\left\lvert\frac{1+x}{1-x}\right\rvert + c$

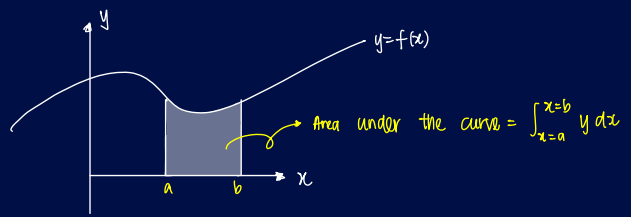
Applications

① Finding equation of curve from gradient function

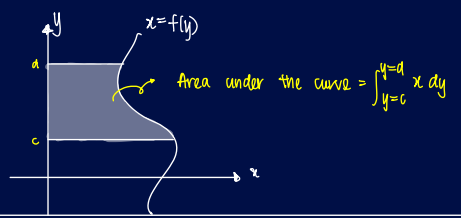


Indefinite integral (no limits)

② Finding area under the curve

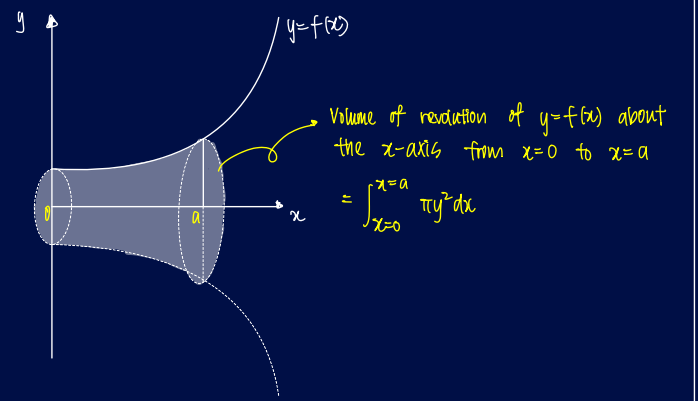


Area under the curve = $\int_{x=a}^{x=b} y\,dx$



Area under the curve = $\int_{y=c}^{y=a} x\,dy$

③ Finding volume of revolution



Volume of revolution of $y=f(x)$ about the x-axis from $x=0$ to $x=a$

$= \int_{x=0}^{x=a} \pi y^2 dx$

Volume of revolution of $x=f(y)$ about the y-axis from $y=a$ to $y=b$

$= \int_{y=a}^{y=b} \pi x^2 dy$ (Try sketching the graph yourself 🥴)